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EVAPOTRANSPIRATION CLIMATONOMY

I. A NEW APPROACH TO NUMERICAL PREDICTION OF MONTHLY EVAPOTRANSPIRATION, RUNOFF, AND SOIL MOISTURE STORAGE

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ABSTRACT

The background and development of a theoretical method to solve the water balance equation for land areas is discussed. A forcing function is considered that is essentially determined by the product of absorbed solar energy multiplied by monthly precipitation; the response function is soil moisture in its month-to-month variations. A very simple parameterization is provided by one nondimensional surface characteristic named the "evaporivity" (which measures the fraction of absorbed insolation utilized during the month in the vaporization of concurrent precipitation) and a characteristic lag-time interval of the order of 2 to 3 mo to express "delayed" evapotranspiration and runoff. The solution is obtained by a closed integration of the water balance equation (rather than employment of regression or correlation methods) and yields a coherent set of data on monthly evapotranspiration, runoff, levels of exchangeable soil moisture, and storage changes. For verification, area averages for the central plains and eastern region of North America are calculated and compared with several years of actual data analyzed and evaluated by Rasmusson. In spite of simplifying tentative assumptions used in the model calculation, the agreement between predicted and observed data is improved in comparison with results of the earlier prediction methods discussed by Rasmusson.

1. INTRODUCTORY REMARKS

By choosing the term "climatonomy," I want to indicate a study of man's physical environment that is significantly more numerically and theoretically oriented than conventional climatology. Etymologically, a parallel can be drawn to a similar distinction between two other branches of atmospheric science, namely "aeronomy" versus "aerology"; the more recently created term "aeronomy" implies a physical-mathematical approach to problems of the upper atmosphere in contrast to the more empirical and statistical methods of conventional aerology. Originally, the word "climatonomy" was coined for my presentation at the 1954 meeting of the American Geophysical Union at Washington, D.C. In his "Review of Climatology 1951–1955," Landsberg (1957) recognized the new term. An etymological explanation and a redefinition were recently provided under the title "Shortwave Radiation Climatonomy," by H. and K. Lettau (1969). It was proposed to use the term "climatonomy" if the existing temporal and spatial variations of basic climatic elements, at any planetary surface, are investigated and explained by a mathematical model that is general enough to permit numerical predictions of climate modification and changes. Characteristic of this approach are theoretical solutions in the form of precisely defined "response functions" or "output cycles" as a physical consequence of a mathematically defined "forcing function," or "energy input cycles" into the environmental system. The "impedances" of the two-layer environmental system (including lowest atmosphere and uppermost submedium or soil strata) will be determined by the nature of forced cycles of those physical processes (and their parameterization) that establish the budget requirements of mass, momentum, and energy at the earth/air interface.

For the climate of any planet, the primary forcing function is insolation, given by the composite of the various cycles (annual to diurnal, including higher harmonics) of solar radiation absorbed by the unit area of a region at the earth's surface. The primary response function is the composite of corresponding cycles of local air and ground temperatures. The physical relationship between forcing and response cycles follows from the transformations of absorbed solar energy which generate also the fluxes that constitute the energy balance of the earth/air interface. These fluxes include terrestrial (long-wave) radiation, latent energy, and sensible heat, the latter being conducted into both air and submedium.

Long-wave radiation and fluxes of sensible heat are relatively straightforwardly coupled with the temperature field on both sides of the earth/air interface; the "partial impedances" contributed by cycles of these fluxes to the total system will be discussed elsewhere. The previously published "Shortwave Radiation Climatonomy" dealt with a parameterization of the forcing function. The present "Evapotranspiration Climatonomy" is concerned with the indirect coupling between the local forcing function and the flux of latent heat in connection with the moisture budget of a continental area. Without precipitation there could be neither soil moisture nor evapotranspiration nor runoff in stream channels, just as there would be no warmth on a planet without insolation from the central sun. With this philosophy it appears logical to develop evapotranspiration climatonomy under due consideration of the composite of the two coexisting forcing functions, namely: 1) insolation, and 2) precipitation; or more poetically: "qualities of sunshine and of rain." From the viewpoint of local climate, the two processes can be considered as relatively independent of each other, both representing external "input," even though from a general circulation viewpoint the two are interdependent. Such interdependency will be incorporated into a more universal climatonomy in a later phase of a continuing development.

With local precipitation as input, and soil moisture variations as output, it is necessary to take into account the two major depleting processes of runoff and evapotranspiration; logically this requires consideration of the conservation equation for mass of water substance in the local column of subsoil. In the course of climatonomical lines of thought, an apparently novel approach to the classical problem of predicting seasonal trends (by monthly mean values) of evapotranspiration, runoff, and storage was developed. The procedure is explained in the following sections. However, it should be emphasized that the present first communication on this subject is restricted to the discussion of a simplified mathematical model that applies best for relatively large land areas or watersheds, and not for the small scale of the microclimate of a site or a field. Correspondingly, the discussion of relatively short periods (diurnal variations and their harmonics or even day-to-day variations, or the characteristic features of a hydrograph describing case studies of a single precipitation event) is postponed. The present discussion will be restricted to monthly averages representing essentially the annual course of hydrometeorological processes. The rigorous derivation of a functional relationship between response and forcing function will be taken as establishing the justification for the somewhat arbitrary title: "Evapotranspiration Climatonomy."

2. LIST OF SYMBOLS, AND BASIC EQUATIONS

For conciseness, the notations to be used for establishing the numerical model of evapotranspiration climatonomy will be grouped genetically as follows:

PHYSICAL (UNIVERSAL) CONSTANTS AND PARAMETERS

σ=Stefan-Boltzmann constant ($0.822 \cdot 10^{-10}$ ly min^{-1} deg^{-1})

ly=unit of one langley=1 cal/cm^2

L=latent heat involved in phase changes of H_2O (for vaporization L=597 cal/gm at 273°K, linearly decreasing with temperature to 580 cal/gm at 303°K; for sublimation, L=677 cal/gm)

DEPENDENT VARIABLES OF PRIMARY IMPORTANCE FOR CLIMATONOMY

T=Kelvin temperatures at and near the ground surface

m=exchangeable moisture contained in a vertical column of soil (mm H_2O, or, alternately, gm H_2O per cm^2) and involved in the hydrologic cycle over land areas

NONDIMENSIONAL CHARACTERISTICS OF PHYSICAL SURFACE STRUCTURE, OF IMPORTANCE FOR CLIMATONOMIC PARAMETERIZATION

a^*=reflectivity (albedo) for short-wave (solar) radiation

ϵ=emissivity for long-wave (terrestrial) radiation

e^*=evaporivity due to insolation (see section 4)

ENERGY FLUX DENSITIES (LANGLEYS PER UNIT OF TIME) AT GROUND LEVEL

G=global radiation=rate at which direct solar plus diffuse sky radiation is received by a horizontal cm^2 at ground level

a^*G=reflected global radiation

LW$\uparrow$=$\epsilon\sigma T^4$=long-wave radiation emitted by the ground surface

LW$\downarrow$=long-wave radiation received by the ground, emitted from overlying atmospheric layers (including dust and cloud strata)

$R=(1-a^*)G-$LW$\uparrow+$LW$\downarrow$=net radiation at ground level

Q=vertical flux of sensible heat into the air, originating at ground level

S=vertical flux of sensible heat into the soil, originating at ground level

LE=vertical flux of latent heat into the air due to phase changes of H_2O substance

MASS FLUX DENSITIES OF WATER SUBSTANCE (MILLIMETERS H_2O PER UNIT OF TIME)

P=precipitation of atmospheric H_2O intercepted by the ground surface

E=evapotranspiration, or vertical flux of H_2O vapor into the air, originating at the ground surface (from bare soil and vegetation)

N=runoff, or rate at which water derived from P reaches stream channels and thus is discharged from the land area (watershed) under consideration

CHARACTERISTIC RATIOS OR NONDIMENSIONALS OF MAJOR IMPORTANCE FOR NUMERICAL MODELING IN CLIMATONOMY

A=Ångström ratio=quotient of effective long-wave radiation to that emitted at ground level

B=Bowen ratio=quotient of fluxes of sensible to latent heat into the air

C=runoff ratio=quotient of runoff to precipitation
D=dryness ratio=quotient of net radiation to heat equivalent (LP) of precipitation

The primary forcing function of planetary climatonomy is given by the cycles of effective short-wave radiation at ground level; that is, the rate at which solar energy (arriving directly along the solar beam as well as indirectly in the form of diffuse light from the sky) can be absorbed by the unit area of ground surface under investigation. This forcing function will be denoted by $F(t)$ since it is a (cyclic) function of time (t); using the notation explained previously,

$$F \equiv (1-a^*)G = F(t). \tag{1}$$

The problem of climatonomy is basically the following: for given cycles of $F(t)$, what is the response function $T(t)$? The solution must be obtained with the aid of the surface energy balance equation, which (with the notation explained previously and using the Ångstrom ratio) can be formulated as

$$F = A\epsilon\sigma T^4 + Q + S + LE. \tag{2}$$

A first simplifying assumption is that "latent" heat refers only to phase changes of water substance; disregarded are photochemical reactions, such as photosynthesis in the chlorophyll-containing tissue of plants exposed to sunlight. A parameterization of such processes could be achieved but will not need to be discussed here.

The desired solution of equation (2) must express the response function in the form of predictable cycles of $T(t)$. Obviously, only the long-wave radiation term explicitly contains temperature, although not linearly but as T^4. It will be shown elsewhere that F-generated cycles of the two fluxes of sensible heat—that is to say, Q and S in equation (2)—contain T implicitly and thus can be related to cycles of $F(t)$. The physical connection between $T(t)$ and $E(t)$, however, is relatively complicated. The introduction of the Bowen ratio (B) is helpful only if this B can be independently estimated, either as a constant or, if variable, as a defined function of energy input as well as of thermal response at the lower boundary of the atmosphere. An example of the latter possibility is the condition at the sea/air interface where there is an unlimited supply of water substance so that the unique-valued dependency of saturation vapor pressure on temperature establishes the necessary link between E and Q in equation (2). A theory for evaporation estimates that utilizes Bowen ratios based on the slope of the saturation pressure versus temperature curve has been proposed by Bryson and Kuhn (1962). However, for unsaturated land, the employment of the Bowen ratio will be meaningful only in terms of the climatic quasi-constants of annual averages. From month to month, also from day to day, the actual state of the soil can fluctuate significantly between extremes of saturation and practically absolute desiccation, and the supply for supporting the evaporative flux in equation (2) must be replenished by precipitation (P). Thus, the Bowen ratio becomes a variable, dependent on preceding precipitation. For the climatonomy of land areas, it is logical that $P(t)$ be taken into account as an additional forcing function while exchangeable soil moisture $m(t)$ assumes the role of a response function in addition to $T(t)$. The problem amounts to simultaneous solutions of equation (2) and the basic balance equation of the mass of water in the soil,

$$P = E + N + dm/dt. \tag{3}$$

For convenience of writing, dm/dt in equation (3) is used to indicate the partial derivative with respect to time. Also, it must be remembered that m expresses a columnar or vertically integrated value of that part of soil moisture that is exchangeable and actively involved in the hydrologic cycle of the land area under consideration. The simultaneous solution of the two balance equations will be facilitated by the fact that E appears as a common term in (2) and (3).

3. ESTIMATE OF ANNUAL AVERAGES OF EVAPOTRANSPIRATION AND RUNOFF

Let an annual average be denoted by the overbar. For a continental region with a stable climate it follows that multiannual means of soil storage terms of both heat and moisture must vanish; that is to say, $\overline{S} = \overline{dm/dt} = 0$. The characteristic ratios defined previously, if formed as quotients of annual means, will be denoted by the asterisk,

$$B^* = \overline{Q}/L\overline{E};\ C^* = \overline{N}/\overline{P};\ D^* = \overline{R}/L\overline{P}. \tag{4}$$

The averaged equations (2) and (3), when solved for $\overline{E}$, yield, under consideration of (4):

$$L\overline{E} = \overline{R} - \overline{Q} = \overline{R}/(1+B^*), \tag{5}$$

and

$$\overline{E} = \overline{P} - \overline{N} = (1-C^*)\overline{P}. \tag{6}$$

The objective is to calculate $\overline{E}$. It may appear paradoxical, but the logical first step is to eliminate $\overline{E}$ in equation (5) with the aid of equation (6), which produces a highly interesting relationship between three nondimensionals,

$$(1+B^*)\cdot(1-C^*) = D^*. \tag{7}$$

Equation (7) has general validity in all cases where S in equation (2), as well as dm/dt in equation (3), vanishes simultaneously. It may appear noteworthy that equation (7) or an equivalent dimensionless expression has apparently not been considered explicitly in recent texts and monographs; this statement can be confirmed by checking the following list of distinguished references: the textbooks entitled *Physical Climatology* by Sellers (1965) and by Landsberg (1958); pertinent monographs by Budyko (1958), Gardner (1965), Thornthwaite and Hare (1965), Landsberg (1957), and Linsley (1951). The author would appreciate any information on possible previous uses of this explicit form of equation (7).

Budyko, in his impressively comprehensive study of world climates in energy and moisture balance terms, however, discusses in great detail his and other investigators' successful attempts to establish a semiempirical relationship between the runoff ratio (C) and the "radiational index of dryness" which is identically the same as the ratio D defined in section 2. Supported by the analysis of actual precipitation, runoff, and net radiation data from a considerable number of watersheds, several possible but similar interpolation formulas have been suggested. Mathematically simplest and reasonably representative appears to be the following, expressed in ratios defined by annual means:

$$C^*=1-\tanh D^*. \tag{8}$$

While equation (7) applies without restrictions if only S and dm/dt are sufficiently small, equation (8) cannot be used in climates where, due to a possible negative value of the annual mean of net radiation, $D^*<0$. Namely, C^* cannot exceed the value of unity, as it would do in equation (8) for negative D^* because the hyperbolic tangent of a negative argument is negative. However, for all positive D^* (which is the norm and not the exception for world climates), equation (8) satisfies the following natural asymptotic conditions. C^* approaches unity in an extremely humid climate (where $\overline{E}$ would approach zero and D^* will be small due to excessive $\overline{P}$), while C^* approaches zero when D^* becomes large in an extremely arid climate (with $\overline{P}$ small and/or $\overline{R}$ large).

For positive D^*, equation (8) can be combined with equation (7) to yield a unique-valued relationship between the three characteristic ratios B^*, C^*, and D^*, as exemplified by the following conjugate sets of triplets:

D^*:	0.00	0.20	0.50	0.70	0.80	0.90	1.00	1.50	2.00	3.00	5.000,
C^*:	1.00	0.80	0.54	0.40	0.34	0.28	0.24	0.10	0.04	0.005	0.000,
B^*:	0.00	0.01	0.08	0.16	0.20	0.26	0.31	0.66	1.07	2.01	4.000.

Given one particular value, for example $C^*=0.263$, there is no need for an interpolation between the numerical values tabulated, because each conjugate triplet can be calculated (with any desired accuracy) by the combination of the two original equations (7) and (8); in the above example of $C^*=0.263$, any standard table of hyperbolic functions yields $D^*=0.945$ whereupon $B^*=0.282=(D^*+C^*-1)/(1-C^*)$. In order to produce dimensional results (that is, the values of $\overline{N}$ and $\overline{E}$, in units of mm/mo, or in./yr, etc.), it is necessary and sufficient that only one input be provided in dimensional form (which will most likely be the annual total or mean of precipitation, $\overline{P}$), in addition to one nondimensional input (for example, C^*, but alternatively the conjugate value of either B^* or D^*).

For the special modeling under consideration, the input normally will be the annual averages of the two climatonomical forcing functions, which are $\overline{P}$ and $\overline{F}$ or, in other words, the two quantitative measures of annual amounts of rain and sunshine. Such input requires, first, the estimation of the mean net radiation $\overline{R}$. This problem can be solved by determining the annual mean of effective radiation, that is $\overline{F}-\overline{R}$, either by using surface emission of terrestrial radiation and a representative Ångström ratio, or with the aid of calculation schemes like the Elsasser chart based on representative aerological data. After $\overline{R}$ has been determined, D^* follows from the given $\overline{P}$, whereupon equation (8) produces C^* and hence, $\overline{N}$ and $\overline{E}$. This is readily supplemented by B^* from equation (7) and by $\overline{Q}=\overline{R}B^*/(1+B^*)$. In summary, if the important relation (7) is valid, the estimate of a coherent set of data on annual evaporation, runoff, and sensible heat flux to the air does not require special parameterization, once a modeling relation like equation (8) has been established. This particular formula is distinguished by the fact that it does not contain any numerical coefficient; however, it appears desirable to test its validity by independent empirical evaluations.

4. NUMERICAL MODEL FOR CALCULATION OF MONTHLY EVAPOTRANSPIRATION, RUNOFF, AND STORAGE

The terms in equation (3) will be considered in physical units of mm(H_2O) per month, which implies that diurnal cycles are averaged out, leaving the annual variation as the main item of interest. As outlined before, the annual cycles of precipitation $P(t)$ and of absorbed insolation $F(t)$ represent the forcing functions, with soil moisture $m(t)$ the response function. A suitable model of climatonomy must express the role of the two depleting processes (evapotranspiration and runoff) as intermediate agents between forcing and response cycles, via water balance requirements.

For the diurnal course of hydrometeorological processes, and for day-to-day variations between individual precipitation events (local storms), the details of the balance between runoff and infiltration, water retention in the soil, vertical movement of H_2O in the soil and to the plant roots, also percolation and drainage, constitute a highly complicated and sometimes confusing system, with significant feedback involved. A wealth of monographs and books has been published on these subjects; it may suffice here to make reference to the collected articles in *Meteorological Monographs*, Vol. 6, on "Agricultural Meteorology," published by the American Meteorological Society (1965), and to the large body of literature quoted there. With respect to details of small-scale and short-period processes, the presently proposed "evapotranspiration climatonomy" may appear oversimplified and will not be a substitute for the special studies reported in the literature. Only within the stated limitations of applicability and for mesoscale to large-scale hydrometeorological processes may the new model and its underlying philosophy be found useful.

Let us assume that two parts contribute to the time series of monthly values of both E and N so that

$$E(t)=E'(t)+E''(t);\ \text{also}\ N(t)=N'(t)+N''(t). \tag{9}$$

The two contributions E' and N' are thought to express a short-term response or a nearly immediate or direct response to the forcing function $P(t)$, while the contributions E'' and N'' express a delayed response. Physically, this distinction implies that the processes denoted by E'' and N'' involve soil moisture that had been received by precipitation of previous months, while the processes denoted by E' and N' are supplied by concurrent precipitation that has just fallen during the same month. In still another alternative form, it can be stated that the lag time involved in the cycling of soil moisture will be less than 1 mo between $P(t)$ and $N'(t)$, $E'(t)$, but more than 1 or 2 mo between $P(t)$ and $E''(t)$, $N''(t)$.

First, consider monthly means of runoff. Let us assume that delayed runoff (N'') varies in direct proportion to soil moisture (m), or

$$N''(t)=\overline{N''}m(t)/\overline{m}, \tag{10}$$

where, as before, the overbar denotes an annual average. Specification of the time series $N'(t)$ offers problems. If there are occasions of saturated soil, or flash floods in connection with unusually heavy rainfall in the region, it must be expected that N' would be significant. Normally N' will be small for sufficiently large land areas unless a monthly mean P-value exceeds a threshold intensity that will depend on the morphology of the basin. The larger the watershed, the smaller the probability that N' may be significant in comparison with N''. For the purpose of a preliminary study, let us disregard N' in comparison with N''; then, $N''(t)$ is practically the same as $N(t)$, or N is given by equation (10).

Next, consider evapotranspiration for which the distinction between immediate (E') and delayed (E'') evapotranspiration has greater significance than for runoff. In analogy to equation (10), we assume for E'' a linear dependency on soil moisture so that

$$E''(\mathrm{t})=\overline{E''}m(t)/\overline{m}. \tag{11}$$

Summing equations (11) and (10) yields the defining equation for a new characteristic (dimensional) parameter

$$N''+E''=(\overline{N''}+\overline{E''})m/\overline{m}=m/t^*, \tag{12}$$

where $t^*=\overline{m}/(\overline{N''}+\overline{E''})$ denotes a characteristic time interval that is most conveniently expressed in units of months. The physical significance of t^* follows from its definition, which is based on annual mean values; t^* can be interpreted as the characteristic "residence time" or "turnover period," which is determined by the quotient of annual mean of exchangeable soil moisture divided by the sum of the combined annual rates of delayed runoff plus delayed evapotranspiration. This definition suggests that t^* could be expected to be a time interval between 2 and 3, or more, months.

As stated before, fundamental for climatonomy is the consideration of a precisely defined forcing function. This concept finds a special application for the estimation of "immediate" evapotranspiration $E'(t)$. Namely, let us assume that $E'(t)$ varies in direct proportion to $F{\cdot}P$, that is, the product of the two existing forcing functions, insolation $F(t)$ times precipitation $P(t)$. Physically, this implies that without either sunshine or rain there will be no "direct" evapotranspiration. For dimensional reasons we consider $1/\overline{F}$ as an additional multiplier whereupon the model assumption requires the introduction of a new characteristic parameter that we propose to name the "evaporivity" e^* of the land surface considered. It enters as the numerical factor of proportionality in the following defining equation:

$$E'(t)=e^*PF/\overline{F};\ \text{also},\ \overline{E'}=(\overline{e^*PF})/\overline{F}. \tag{13}$$

Note that the above e^* is not the same as the "evaporability," which is a dimensional coefficient introduced by Budyko; reference is made to Thornthwaite and Hare (1965). According to the defining equation (13), the evaporivity e^* is a nondimensional measure of the capacity of land surfaces to utilize a portion of solar energy (absorbed during 1 mo) for the evaporation of precipitation that has been received during the same month (or any other specified time interval). In many respects, e^* can be compared with the albedo of continental regions, which, like e^*, is an empirical quantity. Note that e^* may or may not be independent of time or season. In an elementary manner it can be confirmed that $(\overline{PF})/(\overline{F})\cdot(\overline{P})$ may be larger or smaller, or equal to unity; this fraction exceeds unity in a climate with summer rains, and is smaller than unity in a climate with winter rains. Since without irrigation $\overline{E}$ cannot exceed $\overline{P}$, it follows that evaporivity e^* is an always positive fraction that hardly ever will be closer to unity than about 0.8. Indeed, tentative evaluations have suggested that e^* will normally be between 0.4 and 0.8. Its definition suggests that, for a given watershed, the evaporivity e^* should depend on the permeability of the soil, the lushness of vegetation cover, terrain slopes, etc. It will be relatively small for porous soils with "spongy" vegetation, and relatively large on paved or impervious, featureless terrain. An example will be discussed in section 5.

With the aid of the above parameterization and equations (9) to (13), the basic budget equation (3) transforms into

$$P-E'-N'=E''+N''+dm/dt=m/t^*+dm/dt. \tag{14}$$

After taking the annual average of all terms in equation (14), we subtract this average from the original equation (14), and introduce as a convenient abbreviation the symbol p' for the time series $p'=P-E'-N'-(\overline{P-E'-N'})$; this yields

$$p'(t)=(m-\overline{m})/t^*+d(m-\overline{m})/dt. \tag{15}$$

Note that $\overline{p'}=0$, and that the time series $p'(t)$ can be referred to as the reduced version of the "combined" forcing function; it follows from the given values of $P(t)$ and $F(t)$ when e^* is prescribed.

The ordinary differential equation (15) is solved by

$$m-\overline{m}=e^{-t/t^*}\left[\text{const}+\int_0^t e^{t/t^*}p'dt\right], \tag{16}$$

where "const" indicates the integration constant. Note that for given t^* the integration can be performed numerically using any of the standard schemes; the integration constant is determined by the requirement that the "barred" value (that is, the annual mean) of the right-hand side must vanish for a stable climate. Equation (16) characterizes in an exemplary manner the philosophy of climatonomy; namely, a response function $m(t)$ is related to a precisely defined forcing function $p'(t)$, with the aid of a unique-valued equation under employment of mathematically prescribed parameters (in this case e^* and t^*).

The practical verification of the scheme of "soil moisture climatonomy" requires the following sequence of steps:

1) Given $\overline{P}$, calculate annual averages $\overline{E}$ and $\overline{N}$ using procedures outlined in section 3.

2) Assume numerical values for the evaporivity of the watershed under consideration; then, given F and P, calculate monthly values of $e^*PF/\overline{F}=E'$ (according to equation (13)), and obtain the annual mean $\overline{E'}$, whereupon $\overline{E''}$ follows as $\overline{E}-\overline{E'}$.

3) Either assume $N'=0$ (hence, $\overline{N'}=0$, as in the following example of application), or specify numerically what fraction of P (during months of maximum rainfall) constitutes an "immediate" runoff. This step must result in the annual average of N'', which tentatively is taken to equal $\overline{N}$ as derived in step 1).

4) Assume a numerical value for the soil-moisture residence time t^* and obtain the annual average $\overline{m}$ as t^* $(\overline{E''}+\overline{N''})$, using equation (12).

5) On the basis of monthly values for E'—as employed in step 2)—and estimates of N'—as would follow from step 3)—obtain the reduced forcing function, that is, the time series p' in the form of monthly values of $P-E'-N'-(\overline{P-E'-N'})$, and perform numerically the integration prescribed by equation (16); then establish the integration constant, whereupon the time series $m(t)$ is obtained with the aid of $\overline{m}$ as derived in step 4).

6) Calculate the time series dm/dt by subtracting (for each month) the value of m/t^* from $P-E'-N'$, or, alternately, subtract $(m-\overline{m})/t^*$ from p'.

7) Calculate the time series of N'' and E'' with the aid of equations (10) and (11), using soil moisture m from the result of step 5) and the averages determined in steps 2) and 3).

8) Calculate the time series E and N by adding E'' from step 7) to E' from step 2), and N' (if different from zero) to N'' from step 7).

9) Finally, as a check, add the calculated monthly values of dm/dt, E, and N together and compare the result with the original time series $P(t)$. Agreement should be within tolerable rounding-off errors, in view of the basic balance equation (3).

5. EXAMPLE OF APPLICATION

As stated repeatedly, the model of evapotranspiration climatonomy and the calculation of monthly mean water-balance terms will produce best results when applied to a continental region of sufficiently large extent. For the testing of calculations, it is desirable that for such an area the two forcing functions $P(t)$ and $F(t)$ be known, and observational data of all terms of the water balance (that is, monthly means of runoff, evapotranspiration, and storage) be independently available, to be compared with the calculated set of coherent time series of N, E, and m. With respect to data availability and size of the continental area, the work by Rasmusson (1968) appears outstanding; this includes complete time series of monthly water balance terms averaged over several years for the central plains and eastern regions of North America; the area amounts to $64\cdot10^5$ km^2 and includes the watersheds of the Mississippi and St. Lawrence. Rasmusson's data are furthermore distinguished by the feature that monthly evapotranspiration has been evaluated as the vertically integrated divergence of atmospheric vapor transport by large-scale air currents, utilizing the relative dense network of radiosonde stations of the region.

Monthly means of the forcing functions are summarized in table 1. The precipitation values are taken from table 4 in the work by Rasmusson (1968), after being rounded off to the nearest mm/mo. Global radiation for the region (G, ly/day) was estimated independently as a representative area-average, based on selected stations from the report by Löf, Duffie, and Smith (1966); this G-evaluation was performed by Mr. Linde and Mr. Ronberg, both graduate students at the Meteorology Department at Madison, Wis. Albedo values for Rasmusson's region were estimated under consideration of seasonal snow cover conditions and the phenological cycle, essentially based on sample data provided for North America by Kung, Bryson, and Lenschow (1964). The last two lines of table 1 contain the results of straightforward calculations on the basis of the foregoing data.

As the basis for comparison or testing of the model calculations, monthly means of observed water balance terms of the considered region according to Rasmusson (1968, table 4) are summarized in table 2. All values of evapotranspiration (E), runoff (N), soil moisture storing (dm/dt), and soil moisture (m) were rounded off to the nearest mm/mo or mm, and very slightly adjusted so that the sum of the first three lines in table 2 equals the first line (P-value) in table 1, in agreement with the balance equation (3). The fourth line of table 2 represents a smooth time series $(\overline{dm/dt})$ obtained as a three-value running mean of the original dm/dt data.

TABLE 1.—*Central plains and eastern region of North America. Monthly values and representative annual average of: forcing function of precipitation (P, mm/mo); global radiation (G, ly/day); estimated surface albedo (a*, fraction); solar forcing function (F=(1−a*)·G, ly/day); combined forcing function (PF/F̄, mm/mo)*

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Annual mean
P	40	46	55	56	79	87	91	72	73	56	47	45	62
G	187	265	371	466	543	595	589	517	423	312	210	167	398
a^*	.24	.28	.21	.19	.18	.17	.17	.17	.18	.19	.20	.22	.20
F	142	191	293	377	445	494	489	429	347	253	168	130	313
$PF/\overline{F}$	18	28	52	68	112	137	142	99	81	45	25	19	69

TABLE 2.—*Central plains and eastern region of North America. Observed data, after Rasmusson (1968), which constitute the water balance representative of the region. Monthly values and annual average of: evapotranspiration (E, mm/mo); runoff (N, mm/mo); soil moisture storing (dm/dt, mm/mo); smooth time series of soil moisture storing (dm/dt, mm/mo); soil moisture (in mm H_2O) relative to an arbitrary reference level ($m-m_{Aug}$) where m_{Aug} is the undetermined minimum value occurring in August*

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Annual mean
E	23	12	25	40	63	80	93	73	50	40	29	22	46
N	16	17	25	26	23	16	14	13	10	11	11	15	16
dm/dt	1	17	5	−10	−7	−9	−16	−14	13	5	7	8	0
(dm/dt)	8	8	4	−4	−9	−11	−13	−8	1	8	7	5	0
$m-m_{Aug}$	35	52	57	46	40	30	14	0	13	18	25	33	30

In a practical application of the concepts and developments of climatonomy, let us follow the steps outlined at the end of section 4. In step 1), consider that for the region $\overline{P}=62$ mm/mo, according to table 1. Among the various choices of supplementing this information on annual mean forcing function value, let us assume for brevity that the annual mean of stream discharge $\overline{N}=16$ mm/mo (from table 2) is also known. Hence, the runoff ratio for the area equals $C^*=16/62=0.26$, and with this information the empirical Budyko formula (8) yields a dryness ratio of $D^*=0.96$ (since tanh $0.96=0.74=1-0.26$), whereupon the generally valid relation (7) results in a Bowen ratio $B^*=0.30$ for the climate of this region. Knowing the set of ratios (B, C, D), a simple dimensional annual mean (such as $\overline{P}=62$ mm/mo, $L\overline{P}=117$ cal/day) yields the remainder of annual mean energy budget constituents: namely, net radiation as $\overline{R}=D^*L\overline{P}=112$ ly/day, flux of sensible heat into the air $\overline{Q}=B^*\overline{R}/(1+B^*)=26$ ly/day, and heat equivalent of annual mean evaporation $\overline{R}-\overline{Q}=86$ ly/day. The possible comparison of calculated values of $\overline{R}$, $\overline{Q}$, and $\overline{E}$ with independently determined data for the region would permit us to test the validity of equation (8); this test will be outside the scope of our tentative study and will be performed elsewhere.

For step 2), let us assume that the evaporivity of the region is uniform throughout the year and equals $e^*=0.637$. Hence, with $(\overline{PF})/\overline{F}=69$ mm/mo (according to table 1) it follows that the average "immediate" evapotranspiration has an annual average of $\overline{E'}=44$ mm/mo; for the above $\overline{E}=\overline{P}-\overline{N}=46$ mm/mo, only 2 mm/mo remain for the annual average of delayed evapotranspiration $\overline{E''}$.

In step 3), let us assume that for this continent-wide region, all runoff is of the "delayed" type. Hence, $N'=0$ and the annual average $\overline{N''}=\overline{N}=16$ mm/mo, according to developments in step 1).

For step 4), let us postulate that the characteristic residence time interval of soil moisture for the region equals $t^*=3.0$ mo. Consequently, with the sum of the annual averages of delayed processes $\overline{E''}+\overline{N''}=2+16=18$ mm/mo, the annual mean of exchangeable soil moisture of the region follows as $\overline{m}=3.0\times18=54$ mm, or slightly more than 2 in. of water substance. The actual reservoir of ground water is, of course, much deeper since, by definition, m includes only its exchanged portion.

Steps 1) to 4) have involved primarily the calculation of annual averages of energy and water budget constituents of the region. With the following steps we evaluate the time series in the form of one set of values for each of the 12 mo. Step 5) deals with the time series E' (which for a month-to-month variation of e^* would have had to be calculated already in step 2)), and with E' and P (and possibly N') the time series p' is obtained. Results of these two calculations are summarized in the first two lines of table 3. Then, the integration required in equation (16) is performed, which yields the time series of $m-\overline{m}$, hence $(m-\overline{m})/t^*$, and also that of exchangeable soil moisture m; see lines three to five of table 3. For this special example, the integration constant in equation (16) was determined to equal 16 mm H_2O.

Step 6) is the calculation of the time series dm/dt. Rather than approximating this partial derivative by finite differences of the time series $m(t)$, we consider the balance equation (15) and obtain dm/dt for each month as p' minus $(m-\overline{m})/t^*$, that is, the difference between lines two and five in table 3. Values listed in the sixth line of table 3 were calculated in this manner.

When one employs values obtained in the first six steps, the following steps 7) to 9) yield in the straightforward

TABLE 3.—*Results from model of evapotranspiration climatonomy. Calculated monthly data (employing forcing functions and parameters for the central plains and eastern region of North America) of: "direct" evapotranspiration (E', mm/mo); reduced forcing function (p', mm/mo); exchangeable soil moisture (m, also its departure from annual average, ($m-\overline{m}$, mm); rate of soil moisture storing (dm/dt, mm/mo); runoff (N, mm/mo); delayed evapotranspiration (E'', mm/mo); evapotranspiration ($E=E'+E''$, mm/mo); sum of $E+N+dm/dt$ (mm/mo) as "check line"*

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Annual mean
E'	12	18	33	43	71	87	90	63	52	29	16	12	44
p'	10	10	4	−5	−10	−18	−17	−9	3	9	13	15	0
$m-\overline{m}$	16	20	21	13	3	−11	−23	−27	−21	−9	3	14	0
m	70	74	75	67	57	43	31	27	33	45	57	68	54
$(m-\overline{m})/t^*$	5	7	7	4	1	−4	−8	−9	−7	−3	1	5	0
dm/dt	5	3	−3	−9	−11	−14	−9	0	10	12	12	10	0
N	21	22	22	20	17	13	9	8	10	13	17	20	16
E''	2	3	3	2	2	2	1	1	1	2	2	2	2
E	14	21	36	45	73	89	91	64	53	31	18	14	46
$E+N+dm/dt$	40	46	55	56	79	88	91	72	73	56	47	44	62

manner outlined at the end of section 4 the time series of delayed runoff (N'', which equals N for the special case that $N'=0$), delayed evapotranspiration (E''), hence predicted evapotranspiration E as the sum $E'+E''$, and finally $N+E+dm/dt$, which is a "check line" because it should agree (within tolerable rounding-off errors of ± 1 mm/mo) with the time series of precipitation (P) listed in table 1. In summary, table 3 contains all calculated results, that is, the "output" derived with the aid of the model of climatonomy. Table 1 is a documentation of the forcing functions, that is, the "input"; table 2, however, contains exclusively the results derived independently by Rasmusson (1968), that is, a complete set of observed data which can be used to test the results of the calculations.

6. CRITIQUE, AND PLANS FOR FUTURE WORK

The set of data calculated with the model of climatonomy and summarized in table 3 represents a first quantitative solution of the hydrologic balance, with a complete and simultaneous consideration of all terms of equation (3). Calculated are coherent sets of monthly means of area averages of evaporation E, runoff N, exchangeable soil moisture m, and monthly addition or removal from storage, dm/dt. The calculation is based on area averages of two forcing functions, precipitation P and absorbed solar energy F. The fact that all water balance terms are area averages for natural drainage boundaries (that is, river basins or watersheds) is fundamental to hydrology. Rasmusson analyzed results (see table 2) inviting a test by a comparison between corresponding quantities in table 2 and table 3. For a visual comparison, it is useful to enter the calculated results from table 3 into the various illustrations published by Rasmusson (1968), since these graphs already contain comparisons based on the evaluation of formulas and computation schemes proposed earlier by Budyko and also by Thornthwaite. By and large it will be found that the new results of "evapotranspiration climatonomy" appear to simulate relatively closely Rasmusson's curves. The agreement between observed and calculated time series of the primary "response function" of soil moisture $m(t)$, and also of the dominating depletion process of evapotranspiration $E(t)$, becomes even more satisfactory if the calculated E and m series are plotted with a one-half-month lag relative to the observed time series. This could indicate, along the lines of the discussion of immediate versus delayed processes in connection with equations (9), that the lag between $E'(t)$ and the combined forcing function ($P \cdot F$) is relatively small but finite, and not really equal to zero. A suggested lag time of one-half month appears to be appropriate for the "quasi-immediate" processes. No changes in the concept of delayed processes, with a lag time equal to the parameter t^*, or in excess of 2 mo, is necessary.

Incidentally, it should be mentioned that the particular choice of the pair of parameters (that is, $t^*=3.0$ mo and $e^*=0.637$) was based on a preliminary analysis of Rasmusson's empirical data, in the light of climatonomic model concepts. This analysis showed that the model is not very sensitive to t^*, where a variability of ± 20 percent can be tolerated; for e^*, a variability within about ± 5 percent seems to be insignificant. Conditions will be different for other watersheds. The surface characteristic e^* will have to be evaluated for other river basins, where the water balance is known, until a systematic classification according to morphology, soil structure and permeability, vegetation cover, freeze duration, etc. can be made. This procedure will yield classified listings of evaporivity data, comparable to tables presently available, and in general use, for surface reflectivity and emissivity of typical land surfaces. The present employment of a time-independent e^*-value must also be considered as a tentative approximation. In reality, evaporivity at a locality will vary with snow cover, occurrence of partly frozen ground, and the phenological cycle of the growing season, etc. It should be remembered that surface albedo (see a^* in table 1) in temperate climates also has a significant annual course, and is nevertheless a useful and important surface parameter.

In the calculated annual course of soil moisture for the central plains and the eastern regions of North America, the maximum value in early spring as well as the minimum value in late summer appear to be quite realistically produced. The fact that calculated evaporation has nearly the opposite phase as soil moisture illustrates strikingly the dominating effect of "quasi-immediate" evapotranspiration E'; the data also suggest that the efficiency of the process E' is such that most of the rain water is recycled to the atmosphere within about 2 weeks. This, of course, is also evident from the ratio of annual averages, $\overline{E''}/\overline{E'}=$ 2/44, while, naturally, the opposite is true for N'' and N', with N'' dominating. Since N' is near zero for the considered watershed, the calculated N parallels the time series of $m(t)$; the observed data in table 2 show, indeed, that the maxima coincide, while the minimum value of N lags by 1 mo that of the observed m-curve.

With respect to the "delayed" processes E'' and N'', proportionalities of the type of equations (10) and (11) are conventional in hydrology; usually, the restriction is imposed that the proportionality becomes invalid once the soil moisture approaches a state of saturation. This fact was not considered here but could be incorporated relatively easily. In this respect, likewise concerning the incorporation of month-to-month variations of evaporivity, the model of evapotranspiration climatonomy is rather flexible. Refinements will certainly be added in future applications and studies. At the present time, investigations are in progress on climatic data from regions where there are pronounced rainy seasons alternating with many months of dryness, including climates of the Mediterranean type (rainy winters) and of the monsoon character (rainy summers). Obviously, there will be significant differences in the cycle of the combined forcing function $P{\cdot}F$ for these two climates. As indicated in the introduction, the concept of climatonomy is rather broad and not restricted to a special model for the explanation of seasonal soil moisture trends in response to annual forcing cycles represented by the combined action of rain and sunshine. With the knowledge of how to deal basically with the annual variation of the water balance on a continental region, a new road is opened that will lead to a more representative climatonomic model including seasonal temperature trends. Progress in this direction has already been made and will be reported in due time. Apart from the problem of developing climatonomy in general, it may be hoped that the special model of evapotranspiration climatonomy will prove useful for the practical solution of presently existing shortcomings in the parameterization of air-land interaction for numerical modeling of the general circulation of the atmosphere.

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